THEORY OF SOARING FLIGHT

by Wolfgang B. Klemperer

SYNOPSIS OF PART I

Certain observations pertaining to the flight of birds and airplanes appear difficult to reconcile with the orthodox theories of aerodynamics and of power required to sustain flight. Now, if in some of the observed feats of soaring flight of birds, no sign of energy expenditure or consumption is visible, then two explanations may be ventured, one assumes that the bird supplies the energy in some form of muscular effort which is just not visible to the eye, the other concludes that the bird draws energy from an extraneous source, presumably from the atmosphere in which he flies.

Even where there are no vertical wind components the mere existence of regions of different horizontal wind velocities can be utilized by a body which can be guided to fly or reach from one such region into another. The presence of layers of different wind velocities above or adjacent to each other is by no means rare. Gusts accompany most winds.

In studying the various soaring maneuvers and soaring effects it is useful to denote as "dynamic soaring flight" those maneuvers in which inertia effects of the aircraft enter into play as in the utilization of wind layers and gusts.

It is not possible to define the energy available in any dynamic situation in so simple a manner as by the velocity of the vertical wind component of a steady rising current, because the amount of air "affected" depends on the flight "maneuver" as does the power required to execute the same maneuver in calm air.

PART 2

C. Power Required Without Soaring

To express the degree of perfection of a heavier-than-air craft in sustaining a weight aloft at any velocity, a dimensionless quantity can be established as a measure and derived by eliminating the velocity from the equations Lift = Weight and Thrust Horsepower = Drag times Velocity. This dimensionless index will be denoted by

$$\mu = \frac{C_D}{C_L^{3/2}} = P\sqrt{\tfrac{1}{2}\rho\frac{S}{W^3}}$$

where P is the thrust power, ρ the air density, S the wing area, W the weight. Any flight attitude or condition in which the above quantity is a minimum, requires a minimum of power to sustain a given weight at a given air density with a given wing area.

In flight maneuvers of a rhythmic or oscillatory nature a corresponding measure μ of the average power required can be defined by

$$\mu = \int C_D V^3 \frac{dt}{V^0}\sqrt{\frac{C_L^3}{T}}$$

integrated over the time interval T between identical phases of the maneuver cycle. The ratio

$$\frac{\mu}{\mu_o} = \frac{1}{T}\int_0^T \left(\frac{C_D\, V^3}{C_{Do} V_o^3}\right) dt$$

is indicative of the relative power requirement increase of the maneuver. This, however, is a theoretical increase only. In reality the problem always arises as to what thrust power is actually available at any instant. Any disparity between the power required and that applied will primarily affect the flight velocity and secondarily the flight path. Those theoretical maneuvers in which the thrust power applied is always exactly equal to the thrust power required are rather instructive; they may be denoted as "conservative." Their characteristic feature is the reversible exchange between potential energy of altitude and kinetic energy of flight velocity. In the glide, however, no engine power at all is available and gravity is the sole source of motive power.

The glide of least power is the glide of least sinking speed. It is attained at such flight speed $\frac{C_D}{C_R^{3/2}}$ becomes a minimum where C_R is the coefficient of resultant air force related to lift and drag coefficients by $C_R^2 = C_L^2 + C_D^2$. The sinking speed is $V_s = \mu_g V$ where $V = \sqrt{\frac{2W}{S\rho}}$ and $\mu_g = \frac{C_D}{C_R^{3/2}} = \frac{V \cdot \sin\alpha}{V}$

Here α is the glide path slope angle. Since $C_L = C_R \cos\alpha$ it is evident that $\frac{\mu_g}{\mu_{go}} = \cos^{3/2}\alpha$.

This indicates the extent to which the glide of least power consumes less power than the horizontal flight of least power.

Obviously it is of interest to ascertain to what extent other maneuvers departing from straight flight, particularly those of a rhythmic nature, and any that may have a bearing on soaring flight, affect the power requirement by themselves.

Horizontal turns require increased power, obviously, because the aerodynamically generated lift must suffice to make up not only for gravity but also for centrifugal force. The power requirement increase is proportional to $\cos^{-3/2}\beta$ if β represents the correct banking angle; for slow turns this is approximately $1 + {}^{3}/_{4}\beta^2$. Similar formulas apply to the steady helical glide.

If the plane of the path curvature is not horizontal but slant or vertical, matters are more involved. The flight trajectory element at any one instant is essentially governed by an equilibrium between gravity, inertia forces and air forces which can be expressed by a pair of equations, one for the components normal to the flight path, the other parallel to the flight path. For the case of vertical flight maneuvers, these equations can be written in the following, non-dimensional terms referred to in unit weight:

$$\cos\alpha + \frac{V\dot{\alpha}}{g} = C_L\left(\frac{v}{V}\right)^2$$

$$\sin\alpha + \frac{\dot{v}}{g} = -\,C_D\left(\frac{v}{V}\right)^2 + \frac{T}{W}$$

where T/W is the thrust per unit weight assumed to be acting in the direction of flight.

It is instructive to study the power requirement of that particularly simple family of conservative flight maneuvers which F. W. Lanchester named